











Mixed-receiver precise positioning



### **Mixed-Receiver BeiDou Positioning**



BeiDou 2020:

#### 5 GEO, 3 IGSO, 27 MEO

#### **Constellation used for positioning:**





(c) Skyplot (BeiDou)

#### **BeiDou 2013:**

5 GEO, 5 IGSO, 4 MEO

B1:	1561.098 MHz	19.20 cm
B2:	1207.140 MHz	24.83 cm
B3:	1268.520 MHz	23.63 cm

#### **Compare to GPS:**

L1:	1575.420 MHz	19.03 cm
L2:	1227.600 MHz	24.42 cm
L5:	1176.450 MHz	25.48 cm

## Mixed-receiver BeiDou RTK Positioning

[Receivers A-B-C]



Figure 1. Curtin GNSS antennas and receivers used for the 1 km baseline experiment

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1 km Baseline	B1 Ambiguity Success-Rate
CUTO-CUTT (Rx A – Rx A)	0.93
CUT3-CUTJ (Rx <b>B</b> – Rx <b>B</b> )	0.93
CUT1-CUTT (Rx <b>C</b> – Rx <b>A</b> )	0.95

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CUT1-CUTT (Rx <b>C</b> – Rx <b>A</b> )	0.95
CUTO-CUTJ (Rx <b>A</b> – Rx <b>B</b> )	0.0
CUT1-CUTJ (Rx <b>C</b> – Rx <b>B</b> )	0.0

## Existence of mixed-receiver phase-ISTB

(Inter-Satellite-Type-Bias)

Experiment	Receiver Pair	Freq	IGSO-GEO	IGSO-MEO	GEO-MEO
Curtin Exp.	CUTO-CUT1 (Rx <b>A</b> – Rx <b>C</b> )	B1 B2	-0.50		+0.50
20-29 July 2013 (10 day data)	CUTO-CUT2 (Rx <b>A</b> – Rx <b>A</b> )	B1 B2			
(10 day data)	CUTO-CUT3 (Rx <b>A</b> – Rx <b>B</b> )	B1 B2	-0.50		+0.49

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(10 day data)	CUTO-CUT3 (Rx <b>A</b> – Rx <b>B</b> )	B1 B2	-0.50		+0.49

Frequency	Α	С	В
B1	- 0.5	- 0.5	0
B2	0	0.5	0

Table 1. Differential phase ISTB [cyc] between GEO and IGSO/MEO satellites withBthe pivot receiver [1]

## Mixed-receiver BeiDou ISTB is stable over time

Experiment	Receiver Pair	Freq	IGSO-GEO	IGSO-MEO	GEO-MEO
	CUTO-CUT1 (Rx <b>A</b> – Rx <b>C</b> )	B1 B2	-0.50		+0.50
Curtin Exp. 20-29 July	CUTO-CUT2 (Rx <b>A</b> – Rx <b>A</b> )	B1 B2			
2013	CUTO-CUT3 (Rx <b>A</b> – Rx <b>B</b> )	B1 B2	-0.50		+0.49
IGS M-GEX 23-29 Sept 2013	UNX2-UNX3 (Rx <b>B</b> – Rx <b>C</b> )	B1	+0.50		-0.50
	CUTO-CUT1 (Rx <b>A</b> – Rx <b>C</b> )	B1 B2	-0.50		+0.50
Curtin Exp. 17-26 Dec 2013	CUTO-CUT2 (Rx <b>A</b> – Rx <b>A</b> )	B1 B2			
	CUTO-CUT3 (Rx <b>A</b> – Rx <b>B</b> )	B1 B2	-0.50		+0.50
	SPA5-SPA7 (Rx <b>A</b> – Rx <b>B</b> )	B1 B2	-0.50		+0.50

## Phase-ISTB is stable and can be calibrated

1 lun Deceline	B1 Single-epoch Ambiguity Success-Rate				
	Ignore ISTB	Estimate ISTB	Correct for ISTB		
CUTO-CUTT (Rx A – Rx A)	0.93	0.73	0.93		
CUT3-CUTJ (Rx <b>B</b> – Rx <b>B</b> )	0.93	0.73	0.93		
CUT1-CUTT (Rx <b>C</b> – Rx <b>A</b> )	0.95	0.79	0.95		
CUTO-CUTJ (Rx <b>A</b> – Rx <b>B</b> )	0.0	0.68	0.91		
CUT1-CUTJ (Rx <b>C</b> – Rx <b>B</b> )	0.0	0.74	0.94		

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CUTO-CUTT (Rx A – Rx A)	0.93	0.73	0.93		
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Frequency	Α	С	В
B1	- 0.5	0	0
B2	0	0	0

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## Definition of Inter-System Bias (ISB)



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(Absolute) code ISB is defined as the difference of receiver clock A  
and receiver clock B:  

$$ISB_{p,r}^{AB} = cdt_r^B - cdt_r^A = d_r^B - d_r^A - ct_{AB}$$
 (meter)  
ISB = RECEIVER HARDWARE BIAS DIFFERENCE + SYSTEM TIME OFFSET



#### DISB = DOUBLE DIFFERENCE OF RECEIVER HARDWARE BIASES (NOTE: SYSTEM TIME OFFSET IS ELIMINATED)

# Differential ISBs in relative positioning model $\sqrt[q]{q}$

#### Constellation A & B

Differential phase ISB (DISB) is the <u>difference of the absolute ISBs between</u> receiver 1 and <u>r</u>.

## Mixed double-differenced observation model

## Double-differencing w.r.t. pivot satellite per constellation:



# Double-differencing with estimation of unknown DISBs:





MODELS HAVE EXACTLY THE SAME PERFORMANCE, IN TERMS OF AMBIGUITY RESOLUTION & POSITIONING !

Constellation A & B

## Consequence of calibrating the DISBs...



$$\begin{array}{rcl} E(\phi_{1r}^{\mathfrak{S}u}) &=& \rho_{1r}^{\mathfrak{S}u} &+\lambda a_{1r}^{\mathfrak{S}u} \\ E(\phi_{1r}^{\mathfrak{V}}) &=& \rho_{1r}^{\mathfrak{V}} &+\lambda a_{1r}^{\mathfrak{V}} \\ E(p_{1r}^{\mathfrak{S}u}) &=& \rho_{1r}^{\mathfrak{S}u} \\ E(p_{1r}^{\mathfrak{S}v}) &=& \rho_{1r}^{\mathfrak{V}} \end{array}$$

Two pivot satellites (s & q)



$$\begin{split} E(\phi_{1r}^{\mathfrak{su}}) &= \rho_{1r}^{\mathfrak{su}} + \lambda a_{1r}^{\mathfrak{su}} \\ E(\phi_{1r}^{\mathfrak{sq}} - \lambda \Delta ISB_{\phi,1r}^{AB}) &= \rho_{1r}^{\mathfrak{sq}} + \lambda \overline{a}_{1r}^{\mathfrak{sq}} \\ E(\phi_{1r}^{\mathfrak{sv}} - \lambda \Delta ISB_{\phi,1r}^{AB}) &= \rho_{1r}^{\mathfrak{sv}} + \lambda \overline{a}_{1r}^{\mathfrak{sv}} \\ E(\rho_{1r}^{\mathfrak{su}}) &= \rho_{1r}^{\mathfrak{su}} \\ E(p_{1r}^{\mathfrak{su}} - ISB_{p,1r}^{AB}) &= \rho_{1r}^{\mathfrak{su}} \\ E(p_{1r}^{\mathfrak{sv}} - ISB_{p,1r}^{AB}) &= \rho_{1r}^{\mathfrak{sv}} \\ One \text{ pivot satellite; } \text{ with } \overline{a}_{1r}^{\mathfrak{sv}} = a_{1r}^{qv} + \overline{a}_{1r}^{sq} \end{split}$$

 $u_{1r}$ Having an additional satellite, **BETTER PERFORMANCE!** 



## Selected multi-GNSS datasets



#### Co-located MGEX/CUT stations used for our DISB estimation:

Station ID	Receiver type	G	Е	С	J
CUT0 🥿	Trimble NetR9	Y	Y	Y	Y
CUT1 🏈	Septentrio PolaRx4	Y	Y	Y	Ν
CUT2 🏹	Trimble NetR9	Y	Y	Y	Y
CUT3 🍒	Javad TRE-G3TH	Y	Y	Y	Y
USN4 🏈	Septentrio PolaRx4	Y	Y	Ν	Ν
USN5 🍋	Novatel OEM6	Y	Y	Ν	Ν
SIN1	Trimble NetR9	Y	Y	Y	Y
SIN0 🍒	Javad TRE-G3TH	Y	Y	Ν	Y
UNB3 🦕	Trimble NetR9	Y	Y	Y	Ν
UNBD 🍒	Javad TRE-G2T	Y	Y	Ν	Ν
UNBS 🌾	Septentrio PolaRxS	Y	Y	Y	Ν
WTZ2	Leica GR25	Y	Y	Ν	Ν
WTZ3 🍒	Javad TRE-G3TH	Y	Y	Ν	Ν
WTZR	eica GRX1200+GNSS	Y	Y	Ν	Ν

G=GPS; E=Galileo; C=BeiDou; J=QZSS

Estimation of phase and code DISBs:

Zero baseline: 
$$\rho_{1r}^{sq} = 0$$
. Short baseline:  $\rho_{1r}^{sq} =$  known.

All zero baselines (ZB), except:

- UNB3-UNBD/UNBS: 20 m
  - WTZ2-WTZ3: 70 m

which are short baselines (SB)

## BDS+QZSS +Galileo



#### BDS+Galileo+QZSS+GPS



# **Perth (April 20, 2013):** # of satellites for $10^{\circ}$ elevation cut-off angle



System/	Empirical ILS					
Frequency Trimble -Trimble	Success rate % for cut-off [deg.]					
	10	20	25	30	35	40
BDS B1:	96.4	85.4	81.2	63.4	46.7	20.8
L1: Galileo+GPS	81.6	53.9	32.4	17.7	7.5	3.3
E1+L1:	90.3 (86.0)	72.6 (63.9)	53.9 (43.8)	39.8 (29.3)	27.5 (16.7)	19.1 (8.2)
BDS+GPS B1+L1:	98.3	100	100	99.4	96.5	81.7
BDS+ Galileo+ QZSS+GPS:	98.3 (98.3)	100 (100)	100 (100)	100 (99.5)	99.5 (96.7)	91.7 (83.1)

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	10	20	25	30	35	40	
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B1:	96.4	85.4	81.2	63.4	46.7	20.8	
GPS							
L1:	81.6	53.9	32.4	17.7	7.5	3.3	
Galileo+GPS							
E1+L1:	90.3	72.6	53.9	39.8	27.5	19.1	
	(86.0)	(63.9)	(43.8)	(29.3)	(16.7)	(8.2)	
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BDS+	98.3	100	100	100	99.5	91.7	
Galileo+	(98.3)	(100)	(100)	(99.5)	(96.7)	(83.1)	
QZSS+GPS:	(	( )	( )	(	()	()	

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QZSS+GPS:							

System/	Empirical ILS							
Frequency Rx <b>A</b> – Rx <b>A</b>	Success rate % for cut-off [deg.]							
	10	20	25	30	35	40		
BDS								
B1:	96.4	85.4	81.2	63.4	46.7	20.8		
GPS								
L1:	81.6	53.9	32.4	17.7	7.5	3.3		
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QZSS+GPS:	$\bigcirc$							

# Instantaneous RTK float and fixed (correct & incorrect solutions), elevation cut-off angle 35 deg.



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# Positional dilution of precision (PDOP) for different elevation cut-off angles



Mean of the single-epoch PDOP over 4days (April 19-20 and April 29-30, 2013).

System/	Empirical ILS					
Frequency Trimble -Septentrio	Success rate % for cut-off [deg.]					
•	10	20	25	30	35	40
BDS B1: GPS	93.0	80.8	77.0	57.8	44.7	19.3
L1: Galileo+GPS	84.5	60.9	40.3	24.5	11.0	4.1
E1+L1:	92.7 (88.9) <mark>37.4</mark>	78.2 (70.3) <mark>29.9</mark>	60.8 (50.8) 20.7	46.2 (35.5) <mark>12.9</mark>	31.4 (20.3) 5.7	22.6 (10.3) <mark>2.4</mark>
BDS+GPS B1+L1:	98.4	100	100	96.8	91.9	79.9
BDS+ Galileo+ QZSS+GPS:	98.4 (98.4) <mark>97.7</mark>	100 (100) 93.6	100 (100) 88.8	99.8 (97.7) <mark>78.9</mark>	98.7 (93.3) 71.3	89.1 (81.1) <mark>55.8</mark>

System/	Empirical ILS								
Frequency Rx <b>C</b> – Rx <b>A</b>		Success rate % for cut-off [deg.]							
	10	20	25	30	35	40			
BDS B1:	93.0	80.8	77.0	57.8	44.7	19.3			
GPS L1: GalileoGPS	84.5	60.9	40.3	24.5	11.0	4.1			
E1+L1:	92.7 (88.9)	78.2 (70.3)	60.8 <del>(50.8)</del>	46.2 <del>(35.5)</del>	31.4 <u>(20_3)</u>	22.6 (10.3)			
	37.4	29.9	20.7	12.9	5.7	2.4			
B1+L1:	98.4	100	100	96.8	91.9	79.9			
BDS+ Galileo+	98.4 (98.4)	100 <u>(100)</u>	100 (100)	99.8 (97.7)	98.7 (93.3)	89.1 (81.1)			
QZSS+GP8:	97.7	93.6	88.8	78.9	71.3	55.8			

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BDS+ Galileo+ 🤇	98.4 (98.4)	<u>100</u> (100)	<del>100</del> (100)	<del>99.8</del> (97.7)	<u>98.7</u> (93.3)	89.1 (81.1)		
QZSS+GPS:	97.7	93.6	88.8	78.9	71.3	55.8		

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#### Mixed receiver types: elevation cut-off angle 10 deg.



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## Conclusions

#### **Multi-GNSS Mixed-Receiver Positioning**

- Excellent Multi-GNSS precise positioning performance with ISBs known (high cut-off elevations)
- ISBs seem always to be zero for receivers of same manufacturer
- IBSs of **mixed receiver combinations** may not be zero, but if non-zero they are **very stable in time**
- Developed a mixed-receiver ISB Calibration Table
- **Mixed-receiver, multi-GNSS precise positioning impossible** without ISB estimation/calibration