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Satellite Orbit and Clock Error Estimation and Its Applications to GNSS Positioning

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Abstract

In our previous research, we have developed the methods to estimate the VTEC (Vertical Total Electron Content) and to provide the ionospheric delay maps for certain local region such as the sky over Japan. The estimated local models of the VTEC over Japan were also applied to the PPP (Precise Point Positioning), and the positioning accuracy was improved effectively. On the other hand, for the accuracy of PPP, satellite orbit errors and satellite clock errors are also factors should be properly corrected. In this paper, a method to estimate the satellite orbit and clock errors is proposed by extending the algorithms in our previous researches. In the method, the broadcast ephemerides are assumed as the nominal satellite orbits and clock errors. Then satellite position errors and clock errors with respect to the nominal values are modeled by first order Markov processes or Brownian motion processes, and they are estimated by the Kalman filter together with the other unknown quantities such as tropospheric delays, hardware biases and ambiguities. The generated correction information can be easily applied to not only PPP but also any kinds of GNSS positioning algorithms. The proposed method can work by using the GPS data collected by reference stations such as GEONET (Gps Earth Observation NETwork) provided by the Geographical Survey Institute (GSI) of Japan.

1 Introduction

For GNSS (Global Navigation Satellite System) positioning accuracy, effects of ionosphere, troposphere and accuracy of satellite positions and clocks are dominant factors. In our previous research [1-4], we have developed the methods to estimate the ionosphere VTEC (Vertical Total Electron Content) and to provide the ionospheric delay maps for certain local region such as the sky over Japan based on the GNSS regression models (abbreviated by GR models). The estimated local models of the VTEC over Japan were also applied to the PPP (Precise Point Positioning), so that the positioning accuracy was improved effectively. On the other hand, for the accuracy of PPP, satellite orbit errors and satellite clock errors are also factors should be properly corrected.

In this paper, therefore, methods to estimate the satellite orbit and clock errors are proposed by extending the algorithms in [1-4]. The proposed methods to estimate the satellite position are based on the concept of so-called non-dynamic or kinematic orbit determination [5,6]. Namely, the broadcast ephemerides are assumed as the nominal satellite orbits and clock errors. The satellite position errors and clock errors with respect to the nominal values are modeled by first order Markov processes or Brownian motion processes, and then they are estimated by the Kalman filter together with the other unknown quantities such as tropospheric delays, hardware biases and ambiguities. The generated correction information can be easily applied to not only PPP but also any kinds of GNSS positioning algorithms.

The proposed method can work by using the GPS data collected by reference stations with known positions such as GEONET (Gps Earth Observation NETwork) provided by the Geographical Survey Institute (GSI) of Japan.

2 Observation Models

When one receiver receives waves from one GPS satellite, four types of observables are generally observed. They are C/A and P(Y) code pseudoranges and L1 and L2 band carrier phases. Now, assume the receiver k receives the waves from the satellite p, then the observation models of the pseudoranges $\rho^p_{CA,k}(t)$, $\rho^p_{PY,k}(t)$ based on the C/A and P(Y) codes, as well as the L1 and L2 band carrier phases $\varphi^p_{L1,k}(t)$, $\varphi^p_{L2,k}(t)$ are respectively given as follows[7-10]:

$$\begin{aligned}\rho^p_{CA,k}(t) &= r^p_k(t, t-\tau^p_k) + c[\delta t_k(t) - \delta t^p(t-\tau^p_k)] \\ &\quad + \delta I^p_k(t) + \delta T^p_k(t) + \delta b_{CA,k} + \delta b^p_{CA} \\ &\quad + e^p_{CA,k}(t), \end{aligned} \tag{1}$$

$$\begin{aligned}\rho^p_{PY,k}(t) &= r^p_k(t, t-\tau^p_k) + c[\delta t_k(t) - \delta t^p(t-\tau^p_k)] \\ &\quad + \frac{f_1^2}{f_2^2}\delta I^p_k(t) + \delta T^p_k(t) + \delta b_{PY,k} + \delta b^p_{PY} \\ &\quad + e^p_{PY,k}(t), \end{aligned} \tag{2}$$

$$\begin{aligned}\Phi^p_{L1,k}(t) &= \lambda_1\varphi^p_{L1,k}(t)\\ &= r^p_k(t,t-\tau^p_k) + c[\delta t_k(t) - \delta t^p(t-\tau^p_k)]\\ &\quad - \delta I^p_k(t) + \delta T^p_k(t) + \delta b_{L1,k} + \delta b^p_{L1}\\ &\quad + \lambda_1 N^p_{L1,k} + {}_1\varepsilon^p_{L1,k}(t),\end{aligned} \tag{3}$$

$$\begin{aligned}\Phi^p_{L2,k}(t) &= \lambda_2\varphi^p_{L2,k}(t)\\ &= r^p_k(t,t-\tau^p_k) + c[\delta t_k(t) - \delta t^p(t-\tau^p_u)]\\ &\quad - \frac{f_1^2}{f_2^2}\delta I^p_k(t) + \delta T^p_k(t) + \delta b_{L2,k} + \delta b^p_{L2}\\ &\quad + \lambda_2 N^k_{L2,k} + \varepsilon^p_{L2,k}(t),\end{aligned} \tag{4}$$

where c ($\simeq$ 2.99792458 $\times$ 10^8[m/s]) is the speed of light, f_1 and f_2 are L1 and L2 carrier frequencies, i.e. f_1 = 1575.42 [MHz] and f_2 = 1227.60 [MHz] respectively, and λ_1, λ_2 are corresponding wave lengths. $\{\delta b_{CA,k}, \delta b_{PY,k}, \delta b_{L1,k}, \delta b_{L2,k}\}$ are the so-called receiver's hardware biases and $\{\delta b^p_{CA}, \delta b^p_{PY}, \delta b^p_{L1}, \delta b^p_{L2}\}$ are the satellite's hardware biases. $r^p_k(t, t-\tau^p_k)$ is the geometric distance between the receiver k at time t and the satellite p at time $t-\tau^p_k$, where τ^p_k denotes the travelling time of the carrier wave from the satellite p to the receiver k. Namely, $r^p_k(t, t-\tau^p_k)$ can be expressed as

$$\begin{aligned}r^p_k(t) &\equiv r^p_k(t,t-\tau^p_u)\\ &= \{(x_k(t) - x^p(t-\tau^p_k))^2 + (y_(t) - y^p(t-\tau^p_k))^2\\ &\quad +(z_k(t) - z^p(t-\tau^p_k))^2\}^{1/2}\\ &\equiv ||k(t) - s^p(t-\tau^p_k)||,\end{aligned} \tag{5}$$

where $k \equiv [x_k, y_k, z_k]^{\mathrm{T}}$ and $s^p \equiv [x^p, y^p, z^p]^{\mathrm{T}}$ are the known receiver position and the p-th satellite position, respectively, and $||\bullet||$ denotes the Euclidean norm. Further in Eqs. (1)-(4), $\delta I^p_k(t)$ and $\delta T^p_k(t)$ are ionospheric and tropospheric propagation delays or advances, respectively. $\delta t_k(t)$ and $\delta t^p(t-\tau^p_k)$ are the clock errors of the receiver k at time t and the satellite p at time $t-\tau^p_k$. Also $N^p_{l1,u}, N^p_{L2,u}$ are phase ambiguities of L1 and L2 carrier cycles. $e^p_{CA,u}(t), e^p_{PY,u}(t)$ and $\varepsilon^p_{L1,u}(t), \varepsilon^p_{L2,u}(t)$ denote the observation noises.

Eq. (5) shows the geometric relation between the satellite and the receiver. Generally, the most basic method giving the satellite coordinates is to determine them by using the ephemeris broadcast from the satellite. However it contains the satellite orbital errors.

In this paper, the satellite coordinates obtained from the broadcast ephemeris are assumed as the nominal coordinates. Let us define $\hat{s}^p(t, t-\tau^p_k)$ as the nominal position of the satellite p at the time $t-\tau^p_k$. In the followings, the time t and $t-\tau^p_k$ are dropped from each term to simplify the notation. Then, by introducing the satellite position error $\delta s^{p,eph}$, nominal position $\hat{s}^p$ can be expressed as

$$\hat{s}^p = s^p + \delta s^{p,eph}. \tag{6}$$

In other words, the satellite position obtained from the broadcast ephemeris is expressed by the sum of the true position and the ephemeris error.

Similarly to the above, the satellite clock error $\delta t^p(t-\tau^p_k)$ can be also obtained from the ephemeris. However it contains the error. In this paper, the satellite clock error obtained from the ephemeris is assumed as the nominal value, and denoted by $\hat{\delta t}^p$. Therefore $\hat{\delta t}^p$ can be expressed by

$$\hat{\delta t}^p = \delta t^p + \delta t^{p,eph}, \tag{7}$$

where $\delta t^{p,eph}$ denotes the satellite clock error due to the ephemeris error.

Now remark the following formulas of partial derivatives

$$\begin{aligned}\frac{\partial r^p_k}{\partial x^p} &= -\frac{(x_k - x^p)}{r^p_k},\\ \frac{\partial r^p_k}{\partial y^p} &= -\frac{(y_k - y^p)}{r^p_k},\\ \frac{\partial r^p_k}{\partial z^p} &= -\frac{(z_k - z^p)}{r^p_k},\end{aligned} \tag{8}$$

and

$$\frac{\partial r^p_u}{\partial s^p} \equiv \left[\frac{\partial r^p_u}{\partial x^p}, \frac{\partial r^p_u}{\partial y^p}, \frac{\partial r^p_u}{\partial z^p}\right]^{\mathrm{T}}. \tag{9}$$

Then the first order Taylor series approximation of Eq. (5) around the nominal position $\hat{s}^p$ is given by

$$\begin{aligned}r^p_k &\approx r^{\hat{p}}_k - g^{\hat{p}}_k[s^p - \hat{s}^p]\\ &= r^{\hat{p}}_k + g^{\hat{p}}_k\delta s^{p,eph}\end{aligned} \tag{10}$$

where

$$\begin{aligned}r^{\hat{p}}_k &\equiv [r^p_k]_{s^p=\hat{s}^p},\\ g^{\hat{p}}_k &\equiv \left[-\frac{\partial r^p_k}{\partial s^p}\right]^{\mathrm{T}}_{s^p=\hat{s}^p}\\ &= \left[\frac{x_k - \hat{x}^p}{r^{\hat{p}}_k} \quad \frac{y_k - \hat{y}^p}{r^{\hat{p}}_k} \quad \frac{z_k - \hat{z}^p}{r^{\hat{p}}_k}\right].\end{aligned}$$

By substituting δt^p in Eq. (7), and r^p_k in Eq. (10) into Eqs. (1)-(4), we have the following approximated (linearized) measurement equations:

$$\begin{aligned}\tilde{\rho}^p_{CA,k} &\equiv \rho^p_{CA,k} + \hat{\delta t}^p - r^{\hat{p}}_k\\ &= g^{\hat{p}}_k\delta s^{p,eph} + c[\delta t_k(t) + \delta t^{p,eph}]\\ &\quad + \delta I^p_k + \delta T^p_k + \delta b_{CA,k} + \delta b^p_{CA} + e^p_{CA,k},\end{aligned} \tag{11}$$

$$\begin{aligned}\tilde{\rho}^p_{PY,k} &\equiv \rho^p_{PY,k} + \hat{\delta t}^p - r^{\hat{p}}_k\\ &= g^{\hat{p}}_k\delta s^{p,eph} + c[\delta t_k(t) + \delta t^{p,eph}]\\ &\quad + \frac{f_2^2}{f_1^2}\delta I^p_k + \delta T^p_k + \delta b_{PY,k} + \delta b^p_{PY} + e^p_{PY,k},\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{\Phi}^p_{L1,k} &= \Phi^p_{L1,k} + \hat{\delta t}^p - r^{\hat{p}}_k\\ &= g^{\hat{p}}_k\delta s^{p,eph} + c[\delta t_k + \delta t^{p,eph}] - \delta I^p_k + \delta T^p_k\\ &\quad + \delta b_{L1,k} + \delta b^p_{L1} + \lambda_1 N^p_{L1,k} + \varepsilon^p_{L1,k},\end{aligned} \tag{13}$$

$$
\begin{aligned}
\tilde{\Phi}^{p}_{L2,k} &= \Phi^{p}_{L2,k} + \hat{\delta t}^{p} - r^{\hat{p}}_{k} \\
&= g^{\hat{p}}_{k}\delta s^{p,eph} + c[\delta t_k + \delta t^{p,eph}] - \frac{f_1^2}{f_2^2}\delta I^{p}_{k} + \delta T^{p}_{k} \\
&\quad + \delta b_{L2,k} + \delta b^{p}_{L1} + \lambda_1 N^{p}_{L2,k} + \varepsilon^{p}_{L2,k}.
\end{aligned} \tag{14}
$$

Now let n_s be the number of visible satellites at the receiver, and define a block diagonal matrix with the size $(n_s \times 3n_s)$:

$$
G^{\hat{s}}_{D,k} \equiv \mathrm{diag}[\ g^{\hat{1}}_{k},\ g^{\hat{2}}_{k},\ \cdots,\ g^{\hat{n}_s}_{k}\ ], \tag{15}
$$

namely,

$$
G^{\hat{s}}_{D,k} = \begin{bmatrix} g^{\hat{1}}_{k} & O & \cdots & O \\ O & g^{\hat{2}}_{k} & & \vdots \\ \vdots & & \ddots & O \\ O & \cdots & O & g^{\hat{n}_{s,k}}_{k} \end{bmatrix}. \tag{16}
$$

Also define the vectors:

$$
\tilde{\rho}_{CA,k} \equiv \begin{bmatrix} \tilde{\rho}^{1}_{CA,k} \\ \vdots \\ \tilde{\rho}^{n_s}_{CA,k} \end{bmatrix}, \quad \tilde{\Phi}_{L1,k} \equiv \begin{bmatrix} \tilde{\Phi}^{1}_{L1,k} \\ \vdots \\ \tilde{\Phi}^{n_s}_{L1,k} \end{bmatrix},
$$

$$
\delta s^{eph} \equiv \begin{bmatrix} \delta s^{1,eph} \\ \vdots \\ \delta s^{n_s,eph} \end{bmatrix}, \quad \delta t^{eph} \equiv \begin{bmatrix} \delta t^{1,eph} \\ \vdots \\ \delta t^{n_s,eph} \end{bmatrix},
$$

$$
\delta I_k \equiv \begin{bmatrix} \delta I^{1}_{k} \\ \vdots \\ \delta I^{n_s}_{k} \end{bmatrix}, \quad \delta T_k \equiv \begin{bmatrix} \delta T^{1}_{k} \\ \vdots \\ \delta T^{n_s}_{k} \end{bmatrix},
$$

$$
\delta b^{s}_{CA} \equiv \begin{bmatrix} \delta b^{1}_{CA} \\ \vdots \\ \delta b^{n_s}_{CA} \end{bmatrix}, \quad \delta b^{s}_{L1} \equiv \begin{bmatrix} \delta b^{1}_{L1} \\ \vdots \\ \delta b^{n_s}_{L1} \end{bmatrix},
$$

$$
N_{L1,k} \equiv \begin{bmatrix} N^{1}_{L1,k} \\ \vdots \\ N^{n_s}_{L1,k} \end{bmatrix}, \quad e_{CA,k} \equiv \begin{bmatrix} e^{1}_{CA,k} \\ \vdots \\ e^{n_s}_{CA,k}, \end{bmatrix},
$$

$$
\varepsilon_{L1,k} \equiv \begin{bmatrix} \varepsilon^{1}_{L1,k} \\ \vdots \\ \varepsilon^{n_s}_{L1,k}, \end{bmatrix} \quad \delta b_k \equiv \begin{bmatrix} \delta b_{CA,k} \\ \delta b_{PY,k} \\ \delta b_{L1,k} \\ \delta b_{L2,k} \end{bmatrix}.
$$

The vectors $\tilde{\rho}_{PY,k}, \tilde{\Phi}_{L2,k}, \delta b^{s}_{PY}, \delta b^{s}_{PY}, \delta b^{s}_{L2}, N_{L2,k}, e_{PY,k}$ and $\varepsilon_{L2,k}$ are similarly defined.

Then from Eqs. (11)-(14), we have the following vector matrix expression of the observation equation at the receiver k:

$$
y_k = H^{\hat{s}}_{k}\theta_k + v_k \tag{17}
$$

where

$$
y_k \equiv \left[\ \tilde{\rho}^{\mathrm{T}}_{CA,k}, \tilde{\rho}^{\mathrm{T}}_{PY,k}, \tilde{\Phi}^{\mathrm{T}}_{L1,k}, \tilde{\Phi}^{\mathrm{T}}_{L2,k}\ \right], \tag{18}
$$

$$
H^{\hat{s}}_{k} \equiv \begin{bmatrix}
G^{\hat{s}}_{D,k} & \mathbf{1} & I & I & I & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I & O & O & O & O & O \\
G^{\hat{s}}_{D,k} & \mathbf{1} & I & \frac{f_1^2}{f_2^2}I & I & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I & O & O & O & O \\
G^{\hat{s}}_{D,k} & \mathbf{1} & I & -I & I & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & O & I & O & I & O \\
G^{\hat{s}}_{D,k} & \mathbf{1} & I & -\frac{f_1^2}{f_2^2}I & I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & O & O & I & O & I
\end{bmatrix}, \tag{19}
$$

$$
\begin{aligned}
\theta \equiv [&(\delta s^{eph})^{\mathrm{T}}, \delta t_k, (\delta t)^{\mathrm{T}}, (\delta I_k)^{\mathrm{T}}, (\delta T_k)^{\mathrm{T}}, \\
&(\delta b_k)^{\mathrm{T}}, (\delta b^{s}_{CA})^{\mathrm{T}}, (\delta b^{s}_{PY})^{\mathrm{T}}, (\delta b^{s}_{L1})^{\mathrm{T}}, \\
&(\delta b^{s}_{L2})^{\mathrm{T}}, (N_{L1,k})^{\mathrm{T}}, (N_{L2,k})^{\mathrm{T}}]^{\mathrm{T}}
\end{aligned} \tag{20}
$$

$$
v_k \equiv [e^{\mathrm{T}}_{CA,k}, e^{\mathrm{T}}_{PY,k}, \varepsilon^{\mathrm{T}}_{L1,k}, \varepsilon^{\mathrm{T}}_{L2,k}]^{\mathrm{T}}, \tag{21}
$$

and I is the $n_s \times n_s$ identity matrix, $\mathbf{1}$ and $\mathbf{0}$ are the $n_s \times 1$ vectors such that $\mathbf{1} \equiv [1, \cdots, 1]$ and $\mathbf{0} \equiv [0 \cdots 0]^{\mathrm{T}}$.

Eq. (17) is the basic GNSS Regression (GR) equation for the satellite orbit and clock estimation.

2.1 Ionosphere Free Observation

In Eq. (17), by applying the linear transformation called ionosphere free combination, the ionospheric term can be eliminated as follows[5,11].

$$
L_{IF}y_k = L_{IF}H^{\hat{s}}_{k}\theta_k + L_{IF}v_k, \tag{22}
$$

where

$$
L_{IF} \equiv \begin{bmatrix} \mu_1 & \mu_2 & 0 & 0 \\ 0 & 0 & \mu_1 & \mu_2 \end{bmatrix},
$$

$$
\mu_1 = \frac{f_1^2}{f_1^2 - f_2^2}, .\ \mu_2 = \frac{f_2^2}{f_1^2 - f_2^2}.
$$

Then we can reduce the unknown parameters as follows:

$$
\begin{aligned}
&\begin{bmatrix} \mu_1\tilde{\rho}_{CA,k} + \mu_2\tilde{\rho}_{PY,k} \\ \mu_1\tilde{\Phi}_{L1,k} + \mu_2\tilde{\Phi}_{L2,k} \end{bmatrix} \\
&= \left[\begin{matrix} G^{\hat{s}}_{D,k} & \mathbf{1} & I & O & I & \mu_1\mathbf{1} & \mu_2\mathbf{1} & \mathbf{0} & \mathbf{0} \\ G^{\hat{s}}_{D,k} & \mathbf{1} & I & O & I & \mathbf{0} & \mathbf{0} & \mu_1\mathbf{1} & \mu_2\mathbf{1} \end{matrix}\right. \\
&\qquad \left.\begin{matrix} \mu_1 I & \mu_2 I & O & O & O & O \\ O & O & \mu_1 I & \mu_2 I & \mu_1 I & \mu_2 I \end{matrix}\right]\theta_k \\
&\quad + \begin{bmatrix} \mu_1 e_{CA,k} + \mu_2 e_{PY,k} \\ \mu_1\varepsilon_{L1,k} + \mu_2\varepsilon_{L2,k} \end{bmatrix}.
\end{aligned} \tag{23}
$$

Further, define

$$
\begin{aligned}
\delta t_{CA-PY,k} &\equiv \delta t_k + \mu_1\delta b_{CA,k} + \mu_2\delta b_{PY,k} \\
\delta t_{L1-L2,k} &\equiv \delta t_k + \mu_1\delta b_{L1,k} + \mu_2\delta b_{L2,k} \\
\delta t^{eph}_{CA-PY} &\equiv \delta t^{eph} + \mu_1\delta b_{CA} + \mu_2\delta b_{PY} \\
\delta t^{eph}_{L1-L2} &\equiv \delta t^{eph} + \mu_1\delta b_{L1} + \mu_2\delta b_{L2} \\
&\quad + \mu_1 N_{L1,k} + \mu_2 N_{L2,k}
\end{aligned} \tag{24}
$$

then we have

$$\begin{bmatrix} \mu_1\tilde{\rho}_{CA,k} + \mu_2\tilde{\rho}_{PY,k} \\ \mu_1\tilde{\Phi}_{L1,k} + \mu_2\tilde{\Phi}_{L2,k} \end{bmatrix}$$

$$= \begin{bmatrix} G^{\hat{s}}_{D,k} & 1 & 0 & I & O & I \\ G^{\hat{s}}_{D,k} & 0 & 1 & O & I & I \end{bmatrix} \begin{bmatrix} \delta s \\ \delta t_{CA-PY,k} \\ \delta t_{L1-L2,k} \\ \delta t^{eph}_{CA-PY} \\ \delta t^{eph}_{L1-L2} \\ \delta T_k \end{bmatrix} + \begin{bmatrix} \mu_1 e_{CA,k} + \mu_2 e_{PY,k} \\ \mu_1 \varepsilon_{L1,k} + \mu_2 \varepsilon_{L2,k} \end{bmatrix}. \quad (25)$$

2.2 Tropospheric Delay

The tropospheric delay can be considerable for satellites at low elevation angles. Different from the ionospheric delays, the troposphere is not a dispersive medium and causes the same delays for different frequencies. The tropospheric delay is normally represented as having a wet delay and a hydrostatic delay. The wet delay is difficult to model because of local variations in the water-vapor content of the troposphere and accounts for approximately 10% of the tropospheric delay. The hydrostatic delay is relatively well modeled and accounts for approximately 90% of the tropospheric delay.

For the tropospheric delay, in this paper, we apply the following formula of sum of the zenith hydrostatic delay $\delta T_{zh,k}$ and wet delay $\delta T_{zw,k}$ [11,12]. Namely the ZTD (Zenith Total Delay) $\delta T_{z,k}$ can be expressed as follows:

$$\delta T_{z,k} = \delta T_{zh,k} + \delta T_{zw,k}. \quad (26)$$

Then the slant total delay can be expressed by using mapping functions:

$$\delta T^p_k = M^p_{h,k}\delta T_{zh,k} + M^p_{w,k}\delta T_{zw,k}, \quad (27)$$

where

$$M^p_{h,k} = \frac{1}{\sin E^p_{h,k} + \frac{0.00143}{\tan E^p_k + 0.0445}}, \quad (28)$$

$$M^p_{w,k} = \frac{1}{\sin E^p_k + \frac{0.00035}{\tan E^p_k + 0.017}}. \quad (29)$$

$M^p_{h,k}$ and $M^p_{w,k}$ are the mapping functions for the hydrostatic and wet components. E^p_k is the elevation angle of the satellite p at the receiver k.

Furthermore, the zenith wet delay can be expressed by subtracting the zenith hydrostatic delay from the zenith total delay. Then, the slant total delay can be expressed as the following equations by using zenith total delay.

$$\begin{aligned} \delta T^p_k &= M^p_{h,k}\delta T_{zh,k} + M^p_{w,k}(\delta T_{z,k} - \delta T_{zh,k}) \\ &= (M^p_{h,k} - M^p_{w,k})\delta T_{zh,k} + M^p_{w,k}\delta T_{z,k}. \quad (30) \end{aligned}$$

Then, we assume that the zenith hydrostatic delay is measured by the Saastamoinen model:

$$\begin{aligned} \delta\tilde{T}_{zh,k} &= 0.002277(1 + 0.0026\cos 2\phi_k + 0.00028h_k)P_0 \\ &\equiv \delta T_{zh,k} + e_{\delta T_{zh,k}}, \quad (31) \end{aligned}$$

where ϕ_k and h_k denote the latitude and altitude of the position of the receiver k, P_0 denotes the atmospheric pressure and $e_{\delta T_{zh,k}}$ is the error of the model. Also the zenith total delay $\delta T_{z,k}$, is treated as the unknown parameter.

Now, for n_s satellites ($p = 1, 2, \cdots, n_s$), define

$$\begin{aligned} M_{h,k} &\equiv [M^1_{h,k}, \cdots, M^{n_s}_{h,k}]^{\mathrm{T}} \\ M_{w,k} &\equiv [M^1_{w,k}, \cdots, M^{n_s}_{w,k}]^{\mathrm{T}}. \quad (32) \end{aligned}$$

Then, from Eq. (32), the vector δT_k can be expressed as

$$\delta T_{zh,k} = (M_{h,k} - M_{w,k})\delta T_{zh,k} + M_{w,k}\delta T_{z,k}. \quad (33)$$

From Eq. (31), the following relation holds:

$$\delta T_{zh,k} = \delta\tilde{T}_{zh,k} - e_{\delta T_{zh,k}}. \quad (34)$$

Applying Eq. (34) to Eq. (33), then from Eq. (25) we have

$$\begin{bmatrix} \mu_1\tilde{\rho}_{CA,k} + \mu_2\tilde{\rho}_{PY,k} - (M_{h,k} - M_{w,k})\delta\tilde{T}_{zh,k} \\ \mu_1\tilde{\Phi}_{L1,k} + \mu_2\tilde{\rho}_{L2,k} - (M_{h,k} - M_{w,k})\delta\tilde{T}_{zh,k} \end{bmatrix}$$

$$= \left[\begin{array}{c|cc|cc|c} G^{\hat{s}}_{D,k} & 1 & 0 & I & O & M_{w,k} \\ G^{\hat{s}}_{D,k} & 0 & 1 & O & I & M_{w,k} \end{array}\right] \begin{bmatrix} \delta s \\ \delta t_{CA-PY,k} \\ \delta t_{L1-L2,k} \\ \delta t^{eph}_{CA-PY} \\ \delta t^{eph}_{L1-L2} \\ \delta T_{z,k} \end{bmatrix} + \begin{bmatrix} \mu_1 e_{CA,k} + \mu_2 e_{PY,k} - e_{\delta T_{zh,k}} \\ \mu_1 \varepsilon_{L1,k} + \mu_2 \varepsilon_{L2,k} - e_{\delta T_{zh,k}} \end{bmatrix}. \quad (35)$$

Then the simplified expression of Eq. (35) is derived as

$$y_k = \begin{bmatrix} H_{1,k} & H_{2,k} & H_{3,k} & H_{4,k} \end{bmatrix} \begin{bmatrix} \delta s \\ \delta t_{a,k} \\ \delta t^{eph}_a \\ \delta T_{z,k} \end{bmatrix} + v_k, \quad (36)$$

where the matrices $H_{1,k}, \cdots, H_{4,k}$ are defined by the corresponding block matrices separated by the lines in Eq. (35) and

$$y_k \equiv \begin{bmatrix} \mu_1\tilde{\rho}_{CA,k} + \mu_2\tilde{\rho}_{PY,k} - (M_{h,k} - M_{w,k})\delta\tilde{T}_{zh,k} \\ \mu_1\tilde{\Phi}_{L1,k} + \mu_2\tilde{\rho}_{L2,k} - (M_{h,k} - M_{w,k})\delta\tilde{T}_{zh,k} \end{bmatrix},$$

$$v_k \equiv \begin{bmatrix} \mu_1 e_{CA,k} + \mu_2 e_{PY,k} - e_{\delta T_{zh,k}} \\ \mu_1 \varepsilon_{L1,k} + \mu_2 \varepsilon_{L2,k} - e_{\delta T_{zh,k}} \end{bmatrix},$$

$$\delta t_{a,k} \equiv \begin{bmatrix} \delta t_{CA-PY,k} & \delta t_{L1-L2,k} \end{bmatrix}^{\mathrm{T}},$$

$$\delta t^{eph}_a \equiv \begin{bmatrix} (\delta t^{eph}_{CA-PY})^{\mathrm{T}} & (\delta t^{eph}_{L1-L2})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}.$$

2.3 Observation Model for Multiple Reference Stations

From Eq. (36), when the n_k reference stations receive the waves from n_s satellites, totally $2n_s n_k$ of the ionosphere free combinations are observed. The observation equation for the multiple stations can be expressed as follows:

$$y_k = \begin{bmatrix} H_1 & H_2 & H_3 & H_4 \end{bmatrix} \begin{bmatrix} \delta s \\ \delta t_a \\ \delta t_a^{eph} \\ \delta T_a \end{bmatrix} + v, \quad (37)$$

where

$$y \equiv \begin{bmatrix} y_1 \\ \vdots \\ y_{n_k} \end{bmatrix}, \quad v \equiv \begin{bmatrix} v_1 \\ \vdots \\ v_{n_k} \end{bmatrix}, \quad H_1 \equiv \begin{bmatrix} H_{1,1} \\ \vdots \\ H_{1,n_k} \end{bmatrix},$$

$$H_3 \equiv \begin{bmatrix} H_{3,1} \\ \vdots \\ H_{3,n_k} \end{bmatrix}, \quad \delta t_a \equiv \begin{bmatrix} \delta t_{a,1} \\ \vdots \\ \delta t_{a,n_k} \end{bmatrix}, \quad \delta T_a \equiv \begin{bmatrix} \delta T_{z,1} \\ \vdots \\ \delta T_{z,n_k} \end{bmatrix}$$

$$H_2 \equiv \mathrm{diag} \begin{bmatrix} H_{2,1} & \cdots & H_{2,n_k} \end{bmatrix}^{\mathrm{T}},$$

$$H_4 \equiv \mathrm{diag} \begin{bmatrix} H_{4,1} & \cdots & H_{4,n_k} \end{bmatrix}.$$

The numbers of the unknown parameters for n_s satellites and n_k receivers are listed in Table 1.

Table 1: Number of unknown parameters

Unknown vector		No. of components
δs	satellite position error	$3n_s$
δt_a	receivers' clock errors	$2n_k$
δt_a^{eph}	satellites' clock errors	$2n_s$
δT_a	zenith tropospheric delays	n_k
Total number of unknowns		$5n_s + 3n_k$

3 Kalman Filter Formulations

For the estimation of the satellite orbit and clock errors, the Kalman filter is utilized as the estimator. The observation equation for the multiple stations has been shown in Eq. (37). Now we define a matrix

$$H \equiv \begin{bmatrix} H_1 & H_2 & H_3 & H_4 \end{bmatrix}, \quad (38)$$

and a state vector

$$\eta = \begin{bmatrix} (\delta s)^{\mathrm{T}} & (\delta t_a)^{\mathrm{T}} & (\delta t_a^{eph})^{\mathrm{T}} & (\delta T_a)^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \quad (39)$$

Then the observation equation for the Kalman filter can be simplified as

$$y(i) = H(i)\eta(i) + v(i) \quad (40)$$

where the observation epoch i is explicitly shown in the above.

In this paper, the errors of the satellite orbit and clock with respect to those obtained from the ephemerides are modeled by the Brownian motion processes (Wiener processes) or the first order Markov processes. Thus we have

$$\delta s(i+1) = \delta s(i) + w_{\delta s}(i), \quad (41)$$

$$\delta t_a^{eph}(i+1) = \delta t_a^{eph}(i) + w_{\delta t_a^{eph}}(i), \quad (42)$$

or

$$\delta s(i+1) = \alpha \delta s(i) + w_{\delta s}(i), \quad |\alpha| < 1, \quad (43)$$

$$\delta t_a^{eph}(i+1) = \beta \delta t_a^{eph}(i) + w_{\delta t_a^{eph}}(i), \quad |\beta| < 1. \quad (44)$$

The receiver clock errors including receiver hardware biases are modeled by the Brownian motion processes[12,13]:

$$\delta t_a(i+1) = \delta t_a(i) + w_{\delta t_a}(i). \quad (45)$$

The tropospheric zenith delays are modeled by the Brownian motion processes[12,13]:

$$\delta T_a(i+1) = \delta T_a(i) + w_{\delta T_a}(i), \quad (46)$$

where $w_{\delta s}, w_{\delta t_a}, w_{\delta t_a^{eph}}, w_{\delta T_a}$ are independent Gaussian white noises.

Then, from Eqs. (41)-(46), we have the following state equation in a general form

$$\eta(i+1) = F(i)\eta(i) + G(i)w(i), \quad (47)$$

where

$$w \equiv \begin{bmatrix} (w_{\delta s})^{\mathrm{T}} & (w_{\delta t_a})^{\mathrm{T}} & (w_{\delta t_a^{eph}})^{t} & (w_{\delta T_a})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \quad (48)$$

By applying the Kalman filter[14] to the state equation (44) and the observation equation (39), the estimate of η is obtained.

4 Conclusions

In this paper, the method to estimate the errors of the satellite position and the satellite clock correction that are obtained from the broad cast ephemerides has been obtained. The method is based on our previous works such as the algorithms of estimating local models of the VTEC and PPP applied by the GR equations. The method was derived as it can work by using observation data of reference stations for example GEONET in Japan. It is also expected that the correction data generated by the method can be applied to PPP or other positioning algorithms. In the future study, we plan to experiment the proposed method and examine the performance of generated correction data.

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