第3回測位技術振興会研究発表会　2021/9/15 オンライン

# 環状ネットワーク上の分散型Unscented カルマンフィルタ

## Distributed Unscented Kalman Filtering over a Ring Network


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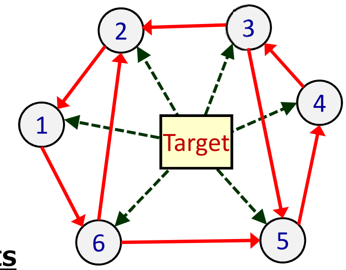
# Introduction

**Distributed State Estimation Problem**

**Each sensor node estimates the state of a target system from the measurements obtained from adjacent sensor nodes.**

Fundamental problem of signal processing in sensor networks.



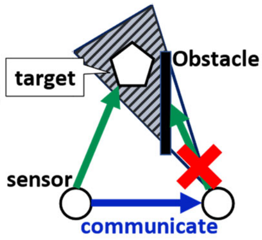

**#** **sensor nodes**

**communication links**

**measurements**

**Merits**

- Improve estimation accuracy by integrating data from each sensor node
- Even if a node is unable to detect the target, it can receive data from other nodes to estimate the state






# Distributed Linear Optimal Filtering

- **R. O.-Saber (2009) Kalman Consensus Filter (KCF)**

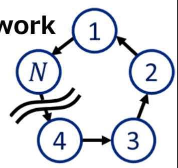
**Assume an observer with embedded consensus control**

$$\hat{x}^i_{t|t} = \hat{x}^i_{t|t-1} + K^i_t\left(y^i_t - C_i x_t\right) + \epsilon P^i_t \sum_{j\in N_i} \left(\hat{x}^j_{t|t-1} - \hat{x}^i_{t|t-1}\right)$$

$\hat{x}^i_{t|t-1}$ : Prediction
$P^i_t$ : Error variance
$K^i_t$ : Kalman gain
$N_i$ : Adjacent sensor set
$\epsilon$ : Consensus weight

**No proof of the optimality of the observer structure**

- **Tsuji, Ohashi, Takaba (IMEKO2021)**




**We derived the truly optimal distributed Kalman filter based on Bayesian inference over a ring network, and verified our distributed filter performs much better than the KCF.**

$$\hat{x}^i_{t|t} = \hat{x}^i_{t|t-1} + K_t^{i(0,0)}\left(y^i_t - C_i\hat{x}^i_{t|t-1}\right) + \cdots + K_t^{i(0,N-1)}\left(y^i_{t-N+1} - C_i\hat{x}^i_{t-N+1|t-1}\right) + L_t^{i(0,0)}\left(\hat{x}^{i+1}_{t|t-1} - \hat{x}^i_{t|t-1}\right) + \cdots + L_t^{i(0,N-1)}\left(\hat{x}^{i+1}_{t-N+1|t-1} - \hat{x}^i_{t-N+1|t-1}\right)$$

$L^i_t = I_n - K^i_t C_i$


[7] R. O.-Saber: " Kalman consensus filter: optimality, stability, and performance", *Proc. of 48th IEEE Conference on Decision and Control*, 2009.
[12] Tsuji, Ohashi, Takaba: " A Bayesian approach to distributed optimal filter over a ring network", *Proc. of 23th IMEKO World Congress*, 2021.

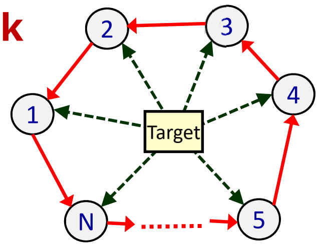
# Introduction

**In this presentation, we extend our result in IMEKO2021 to nonlinear systems by using the Unscented Transformation.**

**Unscented Kalman filter (UKF)**

UKF approximates the distribution of the system state with a Gaussian distribution by using using the **Unscented Transformation.**

**ring network**




Information flow (topology)

measurement

[12] Tsuji, Ohashi, Takaba: " A Bayesian approach to distributed optimal filter over a ring network", *Proc. of 23th IMEKO World Congress*, 2021.



# Problem Formulation

**Sensor Network with $N$ sensor nodes.**

**Target System**

**Conditional PDF**

**State eq.** $\boldsymbol{x}_{t+1} = \boldsymbol{f}(\boldsymbol{x}_t) + \boldsymbol{w}_t$ ⟺ $\boldsymbol{x}_{t+1} \sim \boldsymbol{p}(\boldsymbol{x}_{t+1}|\boldsymbol{x}_t)$

**Measurement eq.** $\boldsymbol{y}_t^i = \boldsymbol{h}_i(\boldsymbol{x}_t) + \boldsymbol{v}_t^i$ ⟺ $\boldsymbol{y} \sim \boldsymbol{p}(\boldsymbol{y}_t^i|\boldsymbol{x}_t)$

$\boldsymbol{x} \in \mathbb{R}^n$ : **the state of the target system to be estimated**

$\boldsymbol{y}^i \in \mathbb{R}^{p_i}$ : **the measurement at node $i$**

$\boldsymbol{Y}_t^i := \{\boldsymbol{y}_1^i, \cdots, \boldsymbol{y}_t^i\}$ : **Set of all measurements up to time $t$ at node $i$**

**Assumption**

$\boldsymbol{w}_t$ : process noise, $\boldsymbol{v}_t^i$ : measurement noise

① The initial distribution of state $x_t$ is $x_0 \sim N(\hat{x}_{0|-1}, P_0)$

② $w_t, v_t^1, \ldots, v_t^N$ are zero mean white Gaussian noise processes such that

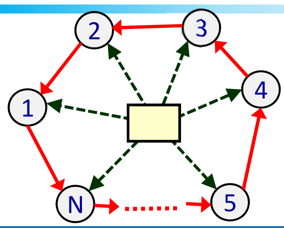
$$\mathbb{E}\left\{\begin{bmatrix} w_t \\ v_t^1 \\ \vdots \\ v_t^N \end{bmatrix}\begin{bmatrix} w_\tau^{\mathrm{T}} & {v_\tau^1}^{\mathrm{T}} & \cdots & {v_\tau^N}^{\mathrm{T}} \end{bmatrix}\right\} = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & R_1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N \end{bmatrix}\delta_{t\tau}$$

$\delta_{t\tau}$ : Kronecker Delta

# Problem Formulation

## Network Structure

**Ring topology with directed information flows (strongly connected)**




← Information flow

←-- measurement

**Assumption ③ (Latency due to data transmission)**

**It takes one unit time-step for the data transmission from one node to its adjacent node.**

**Why ring network ?**

- ✓ Simple enough to intuitively understand the information structure.
- ✓ The performance of the previous KCF due to Saber (2009) significantly deteriorates because of the latency caused by the data relays at each node. ← verified for the linear case (Tsuji, Ohashi, Takaba (2021)).

**Derive the optimal distributed estimation algorithm under the above setting!**



# Bayesian Inference

**Optimal State Estimates at Node $i$**

**Distributed Estimation:**

$$\hat{x}^i_{t|t} = \mathbb{E}\{x_t | Y^i_t, Y^{i+1}_{t-1}, \cdots, Y^N_{t-N+i}, Y^1_{t-N+i-1}, \cdots, Y^{i-1}_{t-N+1}\}$$

due to the data transmission delays between nodes (Assumption ③)

**Centralized Estimation (conventional) :**

$$\hat{x}_{t|t} = \mathbb{E}\{x_t | Y^i_t, Y^{i+1}_t, \cdots, Y^N_t, Y^1_t, \cdots, Y^{i-1}_t\}$$

instantaneous collection of the measurements from all nodes

**To obtain the optimal distributed state estimates,**
**We need to compute the conditional probability density function**

$$p(x_t | Y^i_t, Y^{i+1}_{t-1}, \cdots, Y^N_{t-N+i}, \cdots, Y^{i-1}_{t-N+1})$$

# Distributed Unscented Kalman Filter

The optimal state estimates can be obtained by **recursively** computing the conditional PDFs at the node $i$ based on **the prediction of the adjacent node $i+1$** as well as the measurements at the node $i$ itself.

**Time Update**

$$p(x_t|Y^i_{t-1}, Y^{i+1}_{t-2}, \cdots, Y^{i-1}_{t-N}) = \int p(x_t|x_{t-1})\, p(x_{t-1}|Y^i_{t-1}, Y^{i+1}_{t-2}, \cdots, Y^{i-1}_{t-N})\, dx_{t-1}$$

$i \to i+1$ $\quad$ $t \to t-1$

**Measurement Update**

$$p(x_t|Y^i_t, Y^{i+1}_{t-1}, \cdots, Y^{i-1}_{t-N+1}) = \frac{p(y^i_t, y^i_{t-1}, \cdots, y^i_{t-N+1}|x_t)\, p(x_t|Y^i_{t-N}, Y^{i+1}_{t-1}, \cdots, Y^{i-1}_{t-N+1})}{p(y^i_t, y^i_{t-1}, \cdots, y^i_{t-N+1}|Y^i_{t-N}, Y^{i+1}_{t-1}, \cdots, Y^{i-1}_{t-N})}$$

Measurements up to $N-1$ step obtained by node $i$

Distribution obtained by time update of node $i+1$



# Distributed Unscented Kalman Filter

The optimal state estimates can be obtained by **recursively** computing the conditional PDFs at the node $i$ based on **the prediction of the adjacent node $i+1$** as well as the measurements at the node $i$ itself.

**Time Update**

$$p(x_t|Y^i_{t-1}, Y^{i+1}_{t-2}, \cdots, Y^{i-1}_{t-N}) = \int p(x_t|x_{t-1})\, p(x_{t-1}|Y^i_{t-1}, Y^{i+1}_{t-2}, \cdots, Y^{i-1}_{t-N})\, dx_{t-1}$$

**For a nonlinear system,**
**Approximate these conditional PDFs with Gaussian distributions by applying the Unscented Transform.**

**Measurement Update**

$p(x_t|Y^i_t, Y^{i+1}_{t-1}, \cdots, Y^{i}_{t-}$

Measurements up to $N-1$ step obtained by node $i$

Distribution obtained by time update of node $i+1$

# Distributed Unscented Kalman Filter

**Time update**

$$p(x_t|Y_{t-1}^i, Y_{t-2}^{i+1}, \cdots, Y_{t-N}^{i-1}) = \int p(x_t|x_{t-1})p(x_{t-1}|Y_{t-1}^i, Y_{t-2}^{i+1}, \cdots, Y_{t-N}^{i-1})dx_{t-1}$$

Suppose that the filtered estimate $\hat{x}_{t-1|t-1}^i$ at time $t-1$ at the node $i$ has been obtained.

**The prediction $\hat{x}_{t|t-1}^i$ at time $t-1$ is given by**

$$\hat{x}_{t|t-1}^i = \sum_{k=0}^{2n} W_{F_t}^{i(k)} f\left(x_{t-1|t-1}^{i(k)}\right)$$

$x_{t|t}^{i(k)}$ : σ-points of $p(x_t|Y_t^i, Y_{t-1}^{i+1}, \cdots, Y_{t-N-1}^{i-1})$

$W_{F_t}^{i(k)}$ : weight coefficients

**Prediction error covariance $P_t^i = \mathbb{E}\left\{(x_t - \hat{x}_{t|t-1}^i)(x_t - \hat{x}_{t|t-1}^i)^T\right\}$ is approximated by**

$$P_t^i = \sum_{k=0}^{2n} W_{F_t}^{i(k)} \left(f\left(x_{t-1|t-1}^{i(k)}\right) - \hat{x}_{t|t-1}^i\right)\left(f\left(x_{t-1|t-1}^{i(k)}\right) - \hat{x}_{t|t-1}^i\right)^{\mathrm{T}} + Q$$



# Distributed Unscented Kalman Filter

**Measurement update**

$$p(x_t|Y_t^i, Y_{t-1}^{i+1}, \cdots, Y_{t-N+1}^{i-1}) = \frac{p(y_t^i, y_{t-1}^i, \cdots, y_{t-N+1}^i|x_t)p(x_t|Y_{t-N}^i, Y_{t-1}^{i+1}, \cdots, Y_{t-N+1}^{i-1})}{p(y_t^i, y_{t-1}^i, \cdots, y_{t-N+1}^i|Y_{t-N}^i, Y_{t-1}^{i+1}, \cdots, Y_{t-N+1}^{i-1})}$$

**The filtered estimate $\hat{x}_{t|t}^i$ is given by**

**Past data down to $N-1$**

$$\hat{x}_{t|t}^i = \hat{x}_{t|t-1}^{i+1} + K_t^{i(0,0)}(y_t^i - \hat{y}_{t|t-1}^i) + \cdots + K_t^{i(0,N-1)}(y_{t-N+1}^i - \hat{y}_{t-N+1|t-1}^i)$$

**where $\hat{y}_{t|t}^i$ is the output prediction given by**

$$\hat{y}_{t|t-1}^i = \sum_{k=0}^{2n} W_{H_t}^{i(k)} h_i\left(\hat{x}_{t|t-1}^{i+1(k)}\right)$$

$x_{t|t-1}^{i(k)}$ : σ-points

$W_{H_t}^{i(k)}$ : weight coefficients

**The Kalman gains $K_t^{i(0,k)}$ are computed by using the σ-points, weighting coefficients and the prediction error covariance $P_t^i$.**

**The filtering error covariance $M_t^{i(l,m)} = \mathbb{E}\left\{(x_t - \hat{x}_{t|t-l}^i)(x_t - \hat{x}_{t|t-m}^i)^T\right\}$ can be computed accordingly.**

[12] Tsuji, Ohashi, Takaba: " A Bayesian approach to distributed optimal filter over a ring network", *Proc. of 23th IMEKO World Congress*, 2021.

# Simulation: settings

Verified by MATLAB R2018b

## Tracking an aircraft with 10 radar sensors

### Target motion (state eq.)

$$\begin{bmatrix} x_{t+1} \\ \dot{x}_{t+1} \\ y_{t+1} \\ \dot{y}_{t+1} \\ z_{t+1} \\ \dot{z}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \dot{x}_t \\ y_t \\ \dot{y}_t \\ z_t \\ \dot{z}_t \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t^2 & 0 & 0 \\ \Delta t & 0 & 0 \\ 0 & \frac{1}{2}\Delta t^2 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \frac{1}{2}\Delta t^2 \\ 0 & 0 & \Delta t \end{bmatrix} \begin{bmatrix} u_{x,t} \\ u_{y,t} \\ u_{z,t} \end{bmatrix}$$

### Aircraft Configuration

| | |
|---|---|
| Initial position [m] | $(x,y,z) = (0,0,9000)$ |
| Initial velocity [m/s] | $(v_x,v_y,v_z) = (500,500,15)$ |

### Acceleration input

| | |
|---|---|
| 1~3000 [step] | $(u_{x,t},u_{y,t},u_{z,t}) = (0,0,-1)$ |

### Radar measurements

$$y_t^i = \begin{bmatrix} \sqrt{\Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2} \\ \tan^{-1}\frac{\Delta y_i}{\Delta x_i} \\ \tan^{-1}\frac{-\Delta z_i}{\sqrt{\Delta x_i^2 + \Delta y_i^2}} \end{bmatrix} + v_t^i$$

### Parameter settings

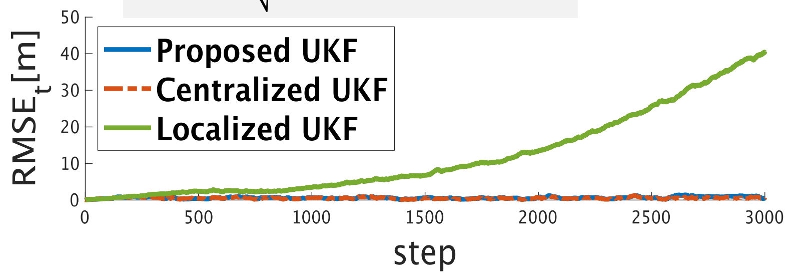
| | |
|---|---|
| Period ($\Delta t$) | 0.01[s] |
| Initial variance ($P_0$) | $I_N$ |
| Standard deviation of measure noise ($R_i$) | $r^i$ : 10[m], $\phi^i,\psi^i$ : 0.05[rad] |
| Standard deviation of process noise ($Q$) | $x,y$ : 10[m/s²], $z$ : 100[m/s²] |
| Hyper parameters of UKF ($\lambda_o$) | Time update : 1, Measure update:0.97 |



# Simulation: results

## RMS Error of entire sensor network at each time instant

$$\mathrm{RMSE}_t = \sqrt{\frac{1}{10}\frac{1}{6}\sum_{n=1}^{6}\sum_{i=1}^{10}\left(x_{n,t} - \hat{x}_{n,t}^i\right)^2}$$




- The estimation accuracy of the proposed method is close to that of the centralized UKF.
- Since radar noises affect more significantly as the aircraft gets distant from a radar, the accuracy of the local UKFs without coordination is degraded, while the proposed method keeps accurate estimation.

# Concluding Remarks

## Summary

- We have briefly reviewed the Bayesian approach to the distributed optimal estimation problem.
- By this approach, we have derived a new distributed UKF algorithm for a nonlinear system.
- We have also verified the effectiveness of our proposed algorithm by the numerical simulation of the tracking filter with 10 radar sensors.

## Future Work

- Extension of the present algorithm to a more general topology of the sensor network.
- Reduction of the computational complexity
- Application to multi-robot SLAM

***Thank you very much for your attention !***